## Problem set 11

Problem 1. Suppose the income elasticity of demand for smartphones is 2 and the income elasticity of demand for house painting is 0.8 . Compare the impact of smartphones and house painting of a recession that reduces consumer income by 5 percent.

Solution. Because smart phones have an income elasticity of demand of 2, we know that they are a normal good, because the coefficient is positive. The same is true for house painting, with a coefficient of 0.8 . However, the income elasticity of house painting is clearly lower than that of smart phones. This means that a reduction of income from a recession will have a greater impact on smart phones than it will on house painting.

Problem 2. After economics class one day, your friend suggests that taxing food would be a good way to raise revenue because the demand for food is quite inelastic. In what sense is taxing food a "good" way to raise revenue? In what sense is it not a "good" way to raise revenue?

Solution. Since the demand for food is inelastic, a tax on food is a good way to raise revenue because it doesn't result in much of a deadweight loss. Thus, taxing food is less inefficient than taxing other things. But it isn't a good way to raise revenue from an equity point of view, since poorer people spend a higher proportion of their income on food, so the tax would hit them harder than it would hit wealthier people.

Problem 3. Some economists argue that private firms will not undertake the efficient amount of basic scientific research. Explain why this is so.

Solution. Since knowledge is a public good, the benefits of basic scientific research are available to many people. The private firm doesn't take this into account when choosing how much research to undertake; it only considers what it will earn, so it produces less basic research than the efficient amount.

Problem 4. Suppose that a market is described by the following supply and demand equations: $P=Q$ and $P=100-Q$.

1. Solve for the equilibrium price and quantity.
2. Suppose that a tax of $\$ 10$ is placed on sellers, so the new supply equation is $P=$ $10+Q$. What happens to the price received by sellers, the price paid by buyers, the quantity sold, and the tax revenue collected by the government?
3. Calculate the deadweight loss from the tax.

## Solution.

1. Set the two equations equal to each other to find the equilibrium quantity. Thus $Q=100-Q$ or, $2 Q=100$, hence $Q=50$. Substitute this value of $Q$ into the demand equation and solve for the equilibrium price. This is $P=50$.
2. Set the new supply equation equal to the demand to find the new equilibrium quantity, so $10+Q=100-Q$, or $2 Q=90$, or $Q=45$. The price the buyers pay is $100-45=\$ 55$. The price the sellers receive is $55-10=\$ 45$. The tax revenue is $10 \times 45=\$ 450$.

3. The deadweight loss from the tax is $1 / 2 \times(50-45) \times 10=\$ 25$.

Problem 5. A profit-maximizing firm in a competitive market is currently producing 100 units of output. It has average revenue of $\$ 10$, average total cost of $\$ 8$, and fixed costs of $\$ 200$.

1. What is its profit?
2. What is its marginal cost?
3. What is its average variable cost?
4. Is the efficient scale of the firm more than, less than, or exactly 100 units?

## Solution.

1. The profit is equal to $(A R-A T C) \times Q$. This is $(10-8) \times 100$, or $\$ 200$.
2. The marginal cost has to equal the marginal revenue at the profit-maximising level of output, hence it is $\$ 10$.
3. Recall that $A T C=A F C+A V C$. If $A T C$ is $\$ 8$, and $A F C$ is $F C / Q$, or $\$ 2$, then $A V C$ must be $\$ 6$.
4. The efficient level of output is less than 100 . The efficient level occurs when the ATC is minimised. This is where $M C=A T C$. Given $M C(\$ 10)$ is greater than $A T C$ (\$8) the firm must be producing at an output level higher than the efficient level.

Problem 6. A company obtains the following information about the demand and production costs of its new product:

Demand: $P=100-Q$
Marginal Revenue: $M R=100-2 Q$
Marginal Cost: $M C=10+Q$

1. Find the price and quantity that maximizes the company's profit.
2. Find the price and quantity that would maximize social welfare.
3. Calculate the deadweight loss from monopoly.

## Solution.

1. Set $M R=M C$ to find the monopoly quantity. Thus, $100-2 Q_{m}=10+Q_{m}$, or $Q_{m}=30$. Substitute this value into the demand equation to find the monopoly price, $P_{m}=100-30=70$.

2. Set $P=M C$ to find the competitive quantity, so $100-Q_{c}=10+Q_{c}$, or $Q_{c}=45$. The competitive price is equal to the marginal cost at the competitive quantity, so $P_{c}=10+45=55$. The competitive price and quantity maximize social welfare.
3. The marginal cost at the monopoly quantity is $10+30=40$. The deadweight loss from monopoly is $1 / 2 \times(45-30) \times(70-40)=225$.

Problem 7. Suppose that changes in bank regulations expand the availability of credit cards so that people need to hold less cash.

1. How does this event affect the demand for money?
2. If the RBA does not respond to this event, what will happen to the interest rate?
3. If the RBA wants to keep the interest rate stable, what should it do?

## Solution.

1. If people need to hold less cash, the demand for money shifts to the left, since there will be less money demanded at any price level.
2. If the RBA does not respond to this event, the shift to the left of the demand for money combined with no change in the supply of money leads to a decline in the interest rate, which means the price level rises, as shown in the figure below.

3. If the Fed wants to keep the interest rate stable, it should reduce the money supply from $M S_{1}$ to $M S_{2}$. This would cause the supply of money to shift to the left by the same amount that the demand for money shifted, resulting in no change in the value of money and the interest rate.

Problem 8. For each of the following events, explain the short-run and long-run effects on output and the price level, assuming policymakers take no action.

1. The stock market declines sharply, reducing consumers' wealth.
2. Yarra Valley grape farms suffer a prolonged period of dry weather.
3. Firms become very optimistic about future business conditions and invest heavily in new capital equipment.

## Solution.

1. When the stock market declines sharply, wealth declines, so the aggregate-demand curve shifts to the left. In the short run, the economy moves from point A to point B, as output declines and the price level declines. In the long run, the short-run aggregate-supply curve shifts to the right to restore equilibrium at point $C$, with unchanged output and a lower price level compared to point A.

2. If Yarra Valley grape farms suffer a prolonged period of dry weather, the grape harvest will be reduced. This is represented in the figure below by a shift to the left in the short-run aggregate-supply curve. The equilibrium changes from point A to point $B$, so the price level rises and output declines.

3. The investment boom will increase the long-run aggregate supply because higher investment today means a larger capital stock in the future, thus higher productivity and output. Thus, the long-run and short-run aggregate-supply curves shift to the right. The economy moves from point A to point B, as output rises and the price level declines. Yes! This is a very difficult problem.


Problem 9. The projected number of persons in different living arrangements in Australia as at 30 June 2016 are as follows.

| Category $(x)$ | Number of persons ('000) |
| :--- | :---: |
| Couple with children (1) | 2473 |
| Couple without children (2) | 2706 |
| One parent male (3) | 185 |
| One parent female (4) | 914 |
| Other (5) | 109 |
| Total | 6387 |

Source: Australian Bureau of Statistics, Household and Family Projections, Australia, 2016-2026, 2012, cat. no. 3236.0, ABS, Canberra

1. Construct the probability distribution of $X$ and present it in graphical form.
2. What is the projected most likely category of living arrangement in Australia in 2016?
3. What is the probability that a randomly selected person is from a 'one parent female' living arrangement category?
4. What is the probability that a person is randomly selected from families living as a 'couple'?
5. What is the expected number and variance of persons in different living arrangements in Australia?

## Solution.

1. Let $X$ be the number of persons in different living arrangements in Australia.

| $X$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $P(X=x)$ | 0.3872 | 0.4237 | 0.0290 | 0.1431 | 0.0171 |

The probability distribution of $X$ is shown below:

2. The most likely category of living arrangement in Australia is couple without children (2).
3. The probability that a randomly selected person is from a 'one parent female' is 0.1431 .
4. $P($ couple $)=P($ Couple with children $)+P($ Couple without children $)=0.3872+$ $0.4237=0.8109$.
5. The following table provides the calculation of mean and variance:

| $x$ | $p(x)$ | $x p(x)$ | $x-\mu$ | $(x-\mu)^{2}$ | $(x-\mu)^{2} p(x)$ |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 0.3872 | 0.387 | -0.980 | 0.959 | 0.371 |
| 2 | 0.4237 | 0.847 | 0.021 | 0.000 | 0.000 |
| 3 | 0.0290 | 0.087 | 1.021 | 1.041 | 0.030 |
| 4 | 0.1431 | 0.572 | 2.021 | 4.082 | 0.584 |
| 5 | 0.0171 | 0.086 | 3.021 | 9.123 | 0.156 |
| $\mu=1.98$ |  |  |  |  | $\sigma^{2}=1.142$ |

Thus, the expected number of persons in different living arrangements in Australia is 1.98 and the variance is 1.142 .

Problem 10. An economist wanted to investigate the relationship between office rents and vacancy rates. Accordingly, he took a random sample of monthly office rents ( $\$^{\prime} 000$ ) and the percentage of vacant office space in 30 different cities. The computer printout is given below.

| SUMMARY OUTPUT |  |
| :--- | ---: |
| Regression Statistics |  |
| Multiple R | 0.5396 |
| R Square | A |
| Adjusted R Square | 0.2658 |
| Standard Error | 2.8732 |
| Observations | 30 |


| ANOVA |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
|  | df | SS | MS | F | Significance F |
| Regression | 1 | 94.94 | 94.94 | 11.50 | 0.0021 |
| Residual | 28 | 231.15 | 8.26 |  |  |
| Total | 29 | 326.09 |  |  |  |


|  | Coefficients | Standard Error | t Stat | P -value |
| :--- | ---: | ---: | ---: | ---: |
| Intercept | B | 1.143 | 18.061 | 0.000 |
| Vacancy rate | -0.304 | C | -3.391 | 0.002 |

1. Using the information from the printout, calculate the value of $\mathrm{A}, \mathrm{B}$, and C .
2. Find the sample regression line and interpret the coefficients.
3. Can we conclude at the $5 \%$ significance level that higher vacancy rates result in lower rents?
4. The economist concluded that higher vacancy rates had caused the office rents to fall. Do you think his conclusion is correct? If not, why?

## Solution.

1. From the ANOVA table, the sum of squared total is $S S T=326.09$, and the sum of squared for regression is $S S R=94.94$. Thus, the value of A is $R^{2}=S S R / S S T=$ $94.94 / 326.09=0.2911$.
From the coefficients table, the standard error of $b_{0}$ is $s_{b 0}=1.143$, and the $t$ statistic is $t_{b 0}=18.061$. It follows that the intercept coefficient $B$ is $b_{0}=t_{0} \times s_{b 0}=18.061 \times$ $1.143=20.64$.
Similarly, the coefficient for vacancy rate is $b_{1}=-0.304$, and the associated $t$ statistic is $t_{b 1}=-3.391$. Thus, the standard error of vacancy rate $C$ is $s_{b 1}=b_{1} / t_{b 1}=$ $-0.304 /-3.391=0.09$.
2. The estimated regression equation is $\hat{y}=20.64-0.304 x$. The slope indicates that for each additional one percentage point increase in the vacancy rate, rents on average decrease by 0.304 thousand dollars. The $y$-intercept is 20.64 , which has no real meaning.
3. The appropriate test is a left one tail test, $H_{0}: \beta_{1}=0, H_{1}: \beta_{1}<0$. The test statistic is $t=-3.391$ and the $p$-value for a one-tail test is $0.002 / 2=0.001$. Therefore, the null hypothesis should be rejected at the $5 \%$ significance level and one could conclude that higher vacancy rates result in lower rents (in a statistical sense).
4. While it seems reasonable to conclude that higher vacancy rates cause lower rents, this conclusion may not be entirely true. It is possible that the rents and vacancy rates are both affected by the state of the economy and the rents fall and vacancy rates rise when the economic condition worsens. It is also theoretically possible that lower rents cause higher vacancy rates because owners hold back on leasing space when the rents are unfavourable. Further analysis would be needed to establish the veracity of this conclusion.
