STATISTICS AND PROBABILITY FOR ENGINEERS

Question # 1: Scores on a certain pediatric sleepiness scale are known to have a mean of 16.5 with a standard deviation of 4.2. You take a random sample of 120 children and administer the sleepiness scale to them. What is the probability that their mean sleepiness scale is between 16.0 and 17.3?

$$
\begin{aligned}
P(16.0 < \bar{x} < 16.7) \quad &= p(-1.30 < z < 2.09) \\
&= p(z < 2.09) - p(z < -1.30) \\
&= 0.9817 - 0.0968 \\
&= 0.8849
\end{aligned}
$$

Question # 2: The average lifetime of a light bulb is 3000 hours with a standard deviation of 696 hours. A simple random sample of 36 bulbs is taken. What is the probability that the average life time in the sample will be between 2670.56 and 2809.76 hours?

$$E(\bar{X}) = \mu = 3000, \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{696}{\sqrt{36}} = 116,$$

$$
\begin{aligned}
&P(2670.56 \le \bar{X} \le 2809.76) \\
&= P\left(\frac{2670.56 - 3000}{116} \le \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{\bar{X} - 3000}{116} \le \frac{2809.76 - 3000}{116}\right) \\
&\approx P(-2.84 \le Z \le -1.64) = 0.0482
\end{aligned}
$$

Question # 3: The average outstanding credit card balance for young couples is \$650 with a standard deviation of \$420. What is the probability that a random sample of 200 young couples have a credit card balance totaling less than \$125,000?

$P\ (T_0 < 125000) = P(z < -0.84) = 0.2005$

Question # 4: Barron's reported that the average number of weeks an individual is unemployed is 17.5 weeks (Barron's, February 18, 2008). Assume that for the population of all unemployed individuals, the population mean length of unemployment is 17.5 weeks and that the population standard deviation is 4 weeks. Suppose you would like to select a random sample of 50 unemployed individuals for a follow-up study. What is the probability that a sample of 50 unemployed individuals will provide a sample mean within 1 week of the population mean?

$p(16.5 < \bar{x} < 18.5) = p(-1.77 < z < 1.77) = p(z < 1.77) - p(z < -1.77) = 0.9616 - 0.0384 = 0.9232$

Question # 5: I eat a breakfast cereal every day. The amount of saturated fat in a serving is normally distributed with mean 25 g and standard deviation 4 g.

If I eat it for a 30-day month, what is the probability that the average daily saturated fat intake for the month was less than 27 g?

$P(\bar{x} < 27) = P(z < 2.74) = 0.9969$

Question # 6: Barron's reported that the average number of weeks an individual is unemployed is 17.5 weeks (Barron's, February 18, 2008). Assume that for the population of all unemployed individuals, the population mean length of unemployment is 17.5 weeks and that the population standard deviation is 4 weeks. Suppose you would like to select a random sample of 50 unemployed individuals for a follow-up study.

What is the probability that a sample of 50 unemployed individuals will provide a sample mean more than 20 weeks?

$p(\bar{x} > 20) = p(z > 4.42) = 1 - p(z < 4.42) = 1 - 1 = 0$

Question # 7: The lifetime of a certain type of battery is normally distributed with mean value 10 hours and standard deviation 1 hour. There are four batteries in a package. What is the probability that the total lifetime of these batteries is at least 42 hours?

$E(T_0) = n\mu = 40 \quad \sigma_{T_0} = \sqrt{4}\sigma = 2$

$p(T_0 > 42) = p(z > 1) = 1 - p(z < 1) = 1 - 0.8413 = 0.1587$

Question # 8: An insurance company is trying to estimate the average number of sick days that full-time food service workers use per year. Assume a population standard deviation to be 2.5 days. The average number of sick days of 20 full-time food service workers use per year is 9 days.

Construct a 95% confidence interval estimate for the population mean number of sick days that full-time food service workers use per year.

95% confidence interval for μ is

$$(\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}})$$

= (9-1.96*0.56, 9+1.96*0.56)

= (9-1.1, 9+1.1) = (7.9, 10.1)

Question # 9: Unoccupied seats on flights cause airlines to lose revenue. Suppose a large airline wants to estimate its average number of unoccupied seats per flight over the past year. To accomplish this, the records of 225 flights are randomly selected and the mean number of unoccupied seats in each flight is 11.596. Assume that the distribution of the number of unoccupied seats in each flight follows a normal distribution with standard deviation 4.103. How large a sample size is required to vary population mean within 0.30 seat of the sample mean with 95% confidence interval?

$$n=\left[\frac{z_{\alpha/2} * \sigma}{E}\right]^2 = (\frac{1.96 * 4.103}{0.30})^2 = 718.58 \approx 719(\text{minimum})$$

Question # 10: A U.S. Travel Data Center survey conducted for *Better Home and Gardens* of 1500 adults found that 39% said that they would take more vacations this year than last year. Find 95% confidence interval for the true proportion of adults who said they will travel more this year and interpret your result.

N = 1500 $\hat{P} = 0.39$

95 % confidence interval for the true proportion of adults who said they will travel more this year is

$$\hat{P} \pm Z_{\alpha/2}\sqrt{\frac{\hat{P}\left(1 - \hat{P}\right)}{n}} = 0.39 \pm 0.025 = (0.365, 0.415)$$

We are 95% confident that the true population proportion of the adult who said they will travel more this year is between 0.365 and 0.415.

<u>Question # 11:</u> Suppose we wish to estimate the proportion of patients in a particular physician's practice with diagnosed osteoarthritis. A random sample of 200 patients is selected and each patient's medical record is reviewed for the diagnosis of osteoarthritis. Suppose that 38 patients are observed with diagnosed osteoarthritis. Compute a 99% confidence interval for the proportion of all patients in the physician's practice diagnosed osteoarthritis and interpret your result.

99% confidence interval for the proportion of all patients in the physician's practice diagnosed Osteoarthritis is

$\hat{P} - Z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}, \hat{P} + Z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$ = [0.19 - (2.575*0.028), 0.19 + (2.575*0.028)]

= [0.119, 0.261]

We are 99% confident that the proportion of all patients in the physician's practice diagnosed Osteoarthritis is between 0.119 and 0.261.

<u>Question # 12:</u> Ten randomly selected automobiles were stopped and the tread depth of the right front tire was measured. The mean was 0.32 inch. and the standard deviation was 0.08 inch. Find the 95% confidence interval of the mean depth. Assume that the variable is approximately normally distributed.

The 95% confidence interval of the mean depth

$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}}, \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}$ = [0.32-2.262*($\frac{0.08}{\sqrt{10}}$), 0.32+2.262*($\frac{0.08}{\sqrt{10}}$)]

= [0.32-0.057, 0.32+0.057]= [0.26, 0.38]

We are 95% confident that the mean depth is between 0.26 and 0.38.

<u>Question # 13:</u> The following data were collected from a random sample of 10 asthmatic children enrolled in a research study and reflect the number of days each child missed school during the past 3 months.

6	12	14	3	2	4	7	8	10	6

Construct and interpret a 95% confidence interval for the population mean number of days each child missed school during the past 3 months.

[Note: $\bar{x}$ = 7.2 and s = 3.88]

95% confidence interval for the population mean is

$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}}, \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}$ = [7.2-2.262*($\frac{3.88}{\sqrt{10}}$), 7.2+2.262*($\frac{3.38}{\sqrt{10}}$)]

= [7.2 – 2.78, 7.2 + 2.78] = [4.42, 9.98]

We are 95% confident that the population mean number of days each child missed school during the past 3 months is between 4.42 and 9.98.

Question # 14: Unoccupied seats on flights cause airlines to lose revenue. Suppose a large airline wants to estimate its average number of unoccupied seats per flight over the past year. To accomplish this, the records of 225 flights are randomly selected and the mean number of unoccupied seats in each flight is 11.596. Assume that the distribution of the number of unoccupied seats in each flight follows a normal distribution with standard deviation 4.103.

Calculate a 99% confidence interval of the population mean number of unoccupied seats and interpret your result.

99% confidence interval for μ is

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = [11.596 - (2.575*0.27), 11.596 + (2.575*0.27)]$$

$$= [11.596 - 0.70, 11.596 + 0.70] = [10.89, 12.30]$$

We are 99% confident that the population mean number of unoccupied seats is between 10.89 and 12.30.

Question # 15: To evaluate the policy of routine vaccination of infants for whooping cough, 339 infants who received their first injection of vaccine were monitored for adverse reaction. Adverse reactions were noted in 69 of these infants. Suppose it is desired to estimate the probability of an adverse reaction to the vaccine to within ±0.03 with 90% confidence. How big a sample of infants receiving their first vaccination should be taken?

$$n = Z_{\alpha/2}^2 \times \frac{\hat{p}\,(1-\hat{p})}{E^2}$$

$$n = 1.645^2 \times \frac{0.2035\,(1-0.2035)}{0.03^2}$$

$$= 487.35$$

$$\approx 488 \text{ (minimum)}$$